Papers

Heave induced pile tension: a simple one-dimensional analysis

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Introduction

Piles are often constructed in soil which subsequently heaves as a result of vertical stress reduction, for example by basement excavation or rising groundwater level. A simple procedure is described for calculating the magnitude of the tension forces induced into such piles and hence the required capacity of tension reinforcement. While the general principles of heave induced tension are widely understood by practising engineers, the authors are not aware of any previous publication which has proposed a one-dimensional soil-pile interaction solution as outlined in this note. This method is able to account for soil stiffness (including non-linear stiffness if appropriate), pile stiffness, pile spacing, the friction properties of the soil-pile interface and time-related effects (eg the rate of swelling). The routine described uses simple assumptions and may easily be programmed. However, it is not so complicated that it cannot be carried out by hand.

This note concentrates specifically on the case of straight-shafted bored piles constructed in soil subjected to heave pressures before the imposition of externally applied loads from buildings or other sources. Building loads usually induce compressive stresses into the pile and hence the critical case for tension will generally occur before the pile experiences such loading. A short note on the extension of the method outlined here to the analysis of cases involving externally applied loads appears at the end of this technical note.

Notation

c_u undrained shear strength
D pile diameter
E'_v effective vertical stiffness of soil under conditions of zero lateral deformation
L pile length
P tensile load in pile
P_{max} the maximum value of P
q value of tensile change in vertical effective stress
Q externally applied load to pile head or pile toe
S pile spacing
T_v Time Factor (consolidation theory)
z depth below pile head level
LTP limiting tension profile
ATP actual tension profile
MFE maximum force envelope
α adhesion factor
δ_s, δ_p soil and pile vertical displacement relative to the neutral point
τ shear stress along the pile-soil interface
τ_{max} soil-pile interface shear strength

Reinforcement percentages are given as a percentage by area of the pile cross-sectional area.

Basic principles of heave induced tension

Consider a pile installed in swelling ground. The soil adjacent to the upper part of the pile moves upwards relative to the shaft, thereby exerting an upward force on the pile. Since the pile remains in equilibrium there must be a downward resulting force of equal magnitude on the pile which must derive from the upward movement of the pile relative to the soil adjacent to the lower part of the pile. The form of shear stress distribution along the pile shaft is thus as shown in ***Figures 1 (a)*** and ***1 (b)*** with a maximum absolute value of shear stress at any level equivalent to the pile-soil interface strength. Summing up the interface strengths (expressed in terms of force per unit length of pile) along the pile from the top produces a cumulative capacity line as shown in ***Figure 1 (c)***; that is to say, a line whose value at any elevation represents the shaft capacity of the pile above that elevation. A similar exercise may be performed summing up the strengths from the bottom. Combining the two cumulative capacity lines gives the limiting tension profile (LTP) as shown in ***Figure 1 (c)*** (Collins, 1953). This LTP represents the maximum theoretical tension force profile in the pile consistent with the assumed strength profile and the equilibrium requirement (assuming no external loading). Also shown in ***Figure 1 (c)*** is the actual tension profile (ATP) for a typical situation, which must in every case lie on or within the LTP.

The point at which the maximum tension force occurs may be considered as a 'neutral point' (Bozozuk, 1972) and corresponds to the plane of zero shear strain in ***Figure 1(a)*** and of zero shear stress in ***Figure 1 (b)***. At this elevation there is no relative displacement between the pile and the soil. The relative displacement increases at points away from the neutral point. The reason that the ATP is not coincident with the LTP is that a finite displacement between the soil and the pile is required for the full interface shear strength to be mobilised, and consequently in the vicinity of the neutral point where the relative soil-pile displacements tend to be small, the tension force on the ATP is less than that on the LTP.

It is possible to define two upper-bound criteria for the maximum tensile force on

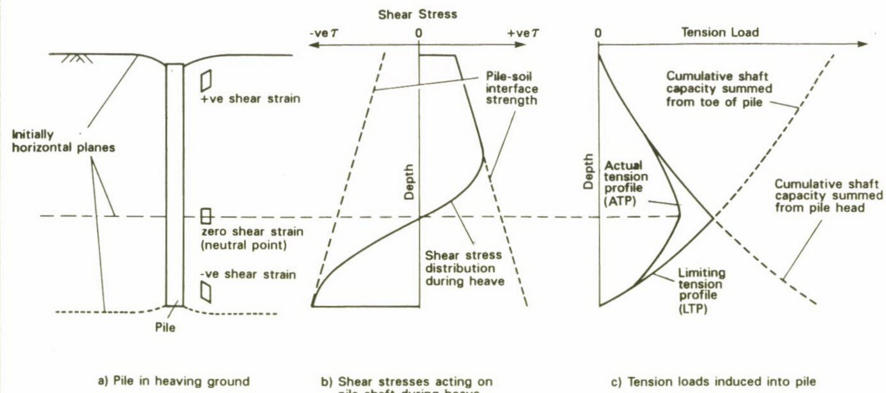

Fig 1. The shear stress and tension load profiles for a pile constructed in heaving ground.

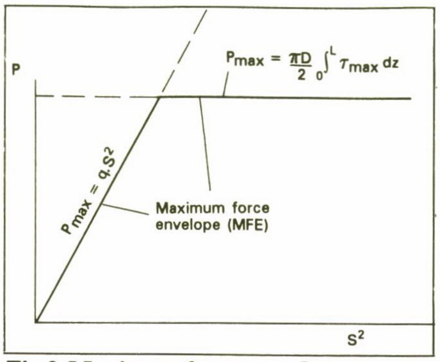

Fig 2. Maximum force envelope.

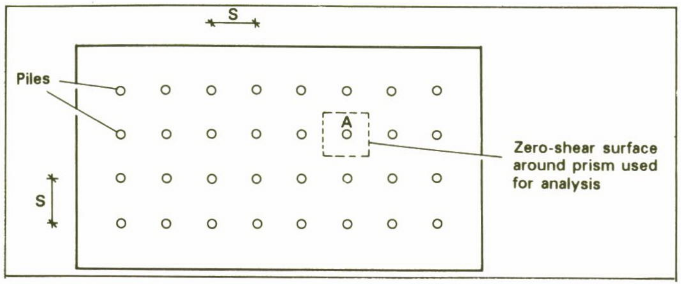

Fig 3. Uniform grid of piles showing prism used for analysis.

the ATP, P_{max}, induced into a pile in swelling ground. These are:

i) P_{max} must be less than or equal to half the total shaft capacity (to maintain equilibrium in the absence of external loading), ie:

$$P_{max} \leqslant 1/2\,[\,\pi D \int_0^L \tau_{max}\, dz\,]$$

ii) P_{max} must be less than or equal to the force which causes the tension. If, for instance, an extensive square grid of piles with spacing S is subjected to uniform surface unloading pressure q (without change in the groundwater regime) the force which generates the heave associated with any given pile is $q.S^2$. Thus:

$$P_{max} \leqslant q.S^2$$

Combining these two inequalities defines a maximum force envelope (MFE) as shown in ***Figure 2.*** For all values of S the tension P at all points along a pile will be less than or equal to the value on the MFE. For widely spaced piles criterion (i) dominates while for closely spaced piles criterion (ii) dominates.

The method of analysis

The analysis described here provides a method for calculating the tension force profile along a pile in heaving ground. It is an interactive one-dimensional soil-pile analysis which takes into account not only the influence of the soil on the pile but also the influence of the pile on the soil. It involves estimating, in the first instance, a tensile load profile in the pile which is in equilibrium. This estimate is then checked for shear stress-displacement compatibility along the pile-soil interface. If the estimate fails to produce a compatible solution, a revised estimate is adopted and the procedure is repeated, iterating until the unique solution for the tensile load profile (ATP) is obtained which is both in equilibrium and satisfies compatibility. The following steps are used in the analysis:

i) Calculate the applied profile of change in tensile effective stress q.

Changes in ground level, surface loading and water regime during construction and events occuring after construction will affect the effective stress profile. For the purposes of this analysis it is necessary to determine the profile of changes in effective tensile stress (denoted q) since it is these which cause heave. In fine-grained soils the change in effective stress will be time-dependent. Simple consolidation calculations enable the profile at a number of times to be calculated so that the heave-induced pile tension can be determined as a function of time.

In this note two distinct profiles of change in effective stress are identified. The first, denoted q, and which is considered here in step (i) refers to changes in effective stress assuming no influence from the piles. It is described as the applied profile of change in tensile effective stress. The second profile, discussed in step (iv) below, is the resultant of the applied profile of changes in tensile stress and the compressive stress induced into the ground as a result of the constraining effect of the piles on the swelling soil.

ii) Estimate the tension force profile (ATP) in the pile.

A first estimate of the ATP may be obtained by choosing a profile somewhat lower than the LTP (*see **Figure 1 c***) and with P_{max} less than the MFE value (*see **Figure 2***).

iii) Divide the pile and soil into layers.

For the purpose of performing the calculations divide the system into a number of layers. The greater the number of layers, the more accurate the solution. Assign to each layer the calculated values of change in tensile stress (q), assumed soil stiffness (E'_v), pile stiffness and soil-pile interface strength (τ_{max}).

iv) Calculate the influence of pile tension on the effective stress changes in the ground.

As noted above, piles tend to constrain the swelling soil. Consider the plan view of a basement with a regular grid of piles with spacing S as shown in ***Figure 3.*** For the purpose of analysing piles in the interior of the piled region a representative pile may be extracted together with its associated soil 'prism' of area S^2. Symmetry considerations indicate that no shear stresses act on the face of the prism so that it can be treated simply as a constant section free standing column of soil with a pile at its core. Piles towards the edge of a group will have equivalent values of S tending towards infinity, as will single piles.

For every unit of load induced into the pile by the soil, the pile must induce an equivalent magnitude of force back into the soil. One simple way of converting this reflected force into a stress is to divide it directly by the prism area S^2. This produces a conservative solution in the sense that the compressive component adjacent to the pile is underestimated using this technique. Using this method, it can be seen that an additional component of compressive stress of magnitude P/S^2 is induced at each level where P is the assumed pile tension at that level. Subtracting the compressive stress component from the change in applied effective stress profile (*see (i) above*) (tensile changes being taken as positive) gives a resultant profile of change in effective stress, $(q - P/S^2)$.

v) Calculate the vertical displacement of the pile and the soil.

Given a tension force profile in the pile (*see (ii) above*) and a change in effective stress profile in the soil (*see (iv) above*) the vertical strain and deformations of each layer of the pile and the soil can be computed from the assumed stiffnesses. By summing up these deformations, a profile of the vertical pile and soil displacements, δ_p and δ_s, can be calculated.

At the neutral point there is zero relative displacement between the soil and the pile; immediately above this point the soil is moving upwards relative to the pile, and immediately below this point the soil is moving downwards relative to the pile. By setting the vertical displacement of both pile and soil to zero at the neutral point, the relative soil-pile shear displacements $(\delta_s - \delta_p)$ along the entire length of the pile can be calculated as shown in ***Figure 4.***

vi) Assume an interface shear stress-displacement relationship and test for compatibility.

Figure 5 shows a possible interface shear stress-displacement relationship. Field data which has been back-calculated from pile tests may be found in the literature for a variety of soil and pile types and arrangements (eg see a number of the contributions in Van Impe (Ed) 1988). For the purpose of the examples given in this note the relationship shown in ***Figure 5*** will be used, as this seems to the authors to

be fairly typical of reported results. Using this shear stress-displacement relationship and the relative shear displacement ($\delta_s - \delta_p$) curve of ***Figure 4*** enables an interface shear stress profile to be computed which can be integrated to give a calculated ATP. This calculated profile can now be compared with the ATP first estimated (*see step (ii)*). Any discrepancy indicates a failure to estimate the profile on the first occasion with sufficient accuracy.

vii) Repeat the process iterating until a consistent solution is produced.

Adopt a compromise ATP between the first estimate and the calculated profiles as a 'revised estimate profile' and repeat the process, iterating until a solution for the ATP is obtained which is both in equilibrium and satisfies stress-displacement compatibility.

Illustrative calculations

In order to illustrate the approach outlined in this note two studies are presented. *Example one* shows the influence of time on the tension forces generated in a pile. It also illustrates the point that even when clay strata are not the predominant soil type, the tensile forces induced may still exceed the pile strength. In *Example two* a series of design charts are produced for a particular stratigraphy (typical of London Clay). These illustrate the influence of pile length, diameter and spacing and soil stiffness on the calculated values of P_{max}.

In these calculations, concrete and steel stiffnesses are assumed to be 20GPa and 210GPa respectively. Creep effects are neglected and the pile is assumed to be reinforced full-length. It is assumed in the sections below that the pile retains an uncracked stiffness but experiences a hairline crack at the elevation of maximum tensile force in the pile so that the steel has to carry all the tension load. The shear stress-displacement relationship used throughout is that shown in ***Figure 5.*** It is assumed for the sake of simplicity that the vertical stress-strain response of the soil is linear. In addition it is assumed that even under long-term drained conditions the soil-pile interface strength for clay is some proportion, α, of the undrained shear strength c_u. The value of α used here is 0.8; this would normally be considered high for pile design but in this case of pile tension a conservative solution requires a high value. The two examples given below were solved by computer, using layers 0.13m and 0.25m thick respectively. The acceptance criterion for the solution was that the input (equilibrium) and output (compatibility) values of tension forces should be within 5% of each other for every soil/pile layer.

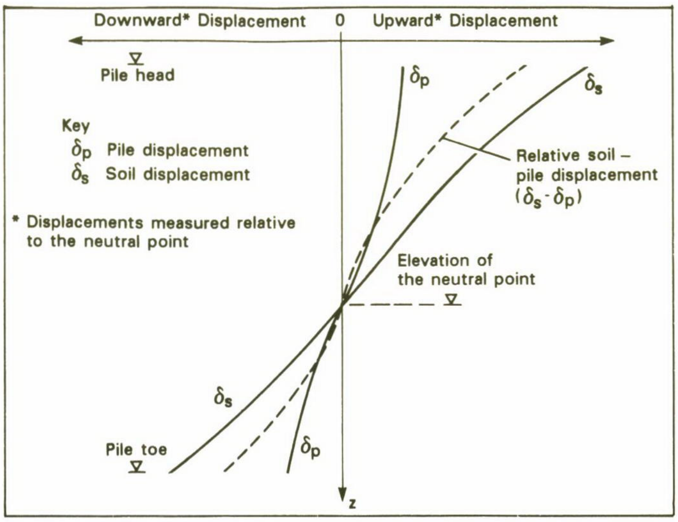

Fig 4. Vertical displacements experienced by soil and pile during heave.

i) Example one:

A single reinforced concrete pile 0.9m in diameter, 20m long with 0.75% longitudinal steel is constructed in the idealised ground profile shown in ***Figure 6(a)***; a layered sand-clay-sand. Initially the ground is subjected to a surcharge of 100kPa which is removed after the pile is built. The vertical effective soil stiffness, E'_v, and maximum soil-pile interface shear stress, τ_{max}, for the granular material are assumed to be $E'_v = (50 + 20z)$MPa and $\tau_{max} = (25 + 10z)$kPa, where z is the depth below the pile head. The shear strength of the clay stratum is assumed to be approximately constant at $c_u = 120$kPa, with stiffness $E'_v = 60$MPa.

The long-term change in tensile effective stress of all layers is q= 100kPa. This occurs very rapidly in the granular material. For the clay, Terzaghi's one-dimensional consolidation theory (eg Terzaghi and Peck 1951) is used to calculate the effective stress changes with time, assuming a free water supply at the top and bottom of the layer.

The computed actual tension profile for a number of time factors for the clay layer, $T_v = 0.1$ to infinity are shown in ***Figure 6(b).*** Also indicated is the LTP and the maximum permissible tension assuming a steel yield strength of $0.35kN/mm^2$. From these computations it seems that for times up to $T_v = 0.12$, 0.75% steel is sufficient to carry the load. After this, external compressive loads (eg building loads at the pile head) will be required to keep the magnitude of the tension forces in the pile below the reinforcement strength. For a soil with a vertical coefficient of consolidation of $5m^2/yr$, $T_v = 0.12$ corresponds to a time of approximately three months for the clay thickness shown, assuming that only vertical drainage occurs.

One matter of some significance which is observed in ***Figure 6*** is that the elevation of the neutral point rises slightly with time. The physical implication of this is that a stress-reversal occurs in the zone between the elevation of the neutral point at $T_v = 0$ and its elevation at $T_v =$ infinity. This is because the soil in this zone initially moves upward relative to the pile, but ends up moving downward relative to the pile. Whenever a non-linear interface stress-displacement relationship is used, this may cause difficulty, as shown in ***Figure 7.*** This figure shows a non-linear relationship and the stress-displacement history OAB experienced by a hypothetical element on the pile-soil interface. If the stress reversal is not accounted for, the interface displacement ($\delta_s - \delta_p$) at B, δ_B, would suggest an interface stress τ_1, whereas in fact τ_2 is the correct value. In order to identify the stress correctly, portions OA and AB of the loading history must be considered separately and in sequence. Since different interface elements will experience stress reversals at different times, a rigorous solution requires a time-incremental solution procedure. In the present case (*Example one*), however, this problem did not arise since the stress-displacement relationship adopted was linear up to 2mm relative displacement; all points in the stress reversal zone were close to the neutral point and hence all relative displacements in this zone were less than 2mm.

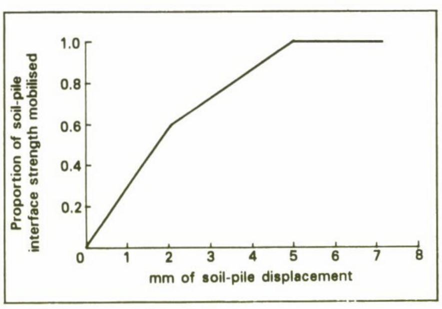

Fig 5. Possible idealised interface shear stress-displacement relationship.

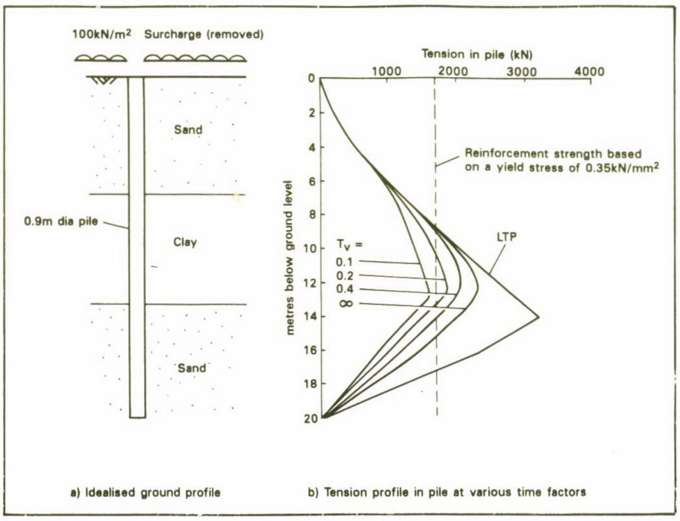

Fig 6. Illustrative example number one.

Consequently no error was caused by performing a one-step computation.

ii) Example two:

Reinforced concrete piles with 1% longitudinal steel reinforcement are constructed in clay with undrained shear strength $c_u = (50 + 5z)$kPa. The vertical stiffness E'_v is assumed to be proportional to c_u. An extensive surface unloading of magnitude q is applied and it is assumed that sufficient time has elapsed since the unloading for pore pressure equilibrium to occur and that as a result all layers of soil experience the same applied tensile change in effective stress, namely q. The analyses assume a single-stage of loading. However, since the soil strains are in fact time-dependent with the rates of strain at any time being a function of depth, a small degree of stress reversal will occur in the general case and hence the solutions given may not be strictly rigorous, as was demonstrated above by reference to ***Figure 7.*** As for *Example one,* however, the error will be slight; its exact magnitude will depend on the prevailing drainage conditions, which in this case have not been specified.

A series of design charts have been produced in ***Figure 8*** showing the predicted value of P_{max} for a range of situations. Two values of unloading, q=50kPa and 100kPa are considered in ***Figures 8a and b*** respectively. Three pile lengths, L=10m, 20m and 30m are considered in parts (i), (ii) and (iii) respectively of ***Figures 8a and b.*** The axes adopted (P_{max}/D vs S^2/D) in these charts were chosen so that the MFE was represented by a simple and unique bi-linear relationship for all pile diameters.

As an example of the use of the charts consider the case of 1.2m diameter piles, 30m long set out on an extensive square grid of 8m installed in a basement which experiences about 5m soil removal without affecting the groundwater regime (ie q is about 100kPa). Consider a pile in the interior of the piled area for which S=8m, giving (S^2/D)=53m. Assume that E'_v is 500c_u. From ***Figure 8b(iii)*** and interpolating between E'_v=200c_u and E'_v=800c_u gives P_{max}/D=3375kN/m or P_{max}=4050kN which is equivalent to a tensile stress in the steel of 0.358kN/mm^2 if there is 1% reinforcement. The designer must consider whether this stress is excessive, or if in view of the conservative assumptions made (eg full dissipation of pore pressures) it is acceptable.

A number of general features emerge from these analyses. The following are examples:

i) the importance of the MFE, which bounds all the results, is apparent.
ii) the assumed soil stiffness has a marked influence on P_{max}.
iii) closely spaced piles are strongly influenced by the sloping portion of the MFE and are therefore very dependent on the magnitude of unloading q, while more widely spaced piles are less dependent on q.

iv) for any given spacing parameter S^2/D the value of P_{max}/D is to a first approximation independent of the pile diameter especially for the 10m and 20m long piles; hence the load in the pile is proportional to the diameter and the percentage reinforcement required is inversely proportional to the pile diameter.

Comparison with field data and an elastic solution incorporating pile-soil slip

Field observations of heave-induced tension are rare in the literature. One case history reported by Donaldson (1967) and analysed by Poulos and Davies (1979) concerns a 230mm diameter, 9.15m long pile in expansive soil in dry southern African conditions which was made to heave by inundating the area around the pile with water. Poulos and Davis assumed soil stiffness and soil-pile interface strength profiles. They used an elastic analysis with interface slip when the interface strength was attained.

In order to compare the present method with an elastic solution incorporating soil-pile slip, the soil properties assumed by Poulos and Davis (which may be found in their book) were input into the present analysis. Since they assumed that soil remote from the pile would heave 9mm, a change in tensile effective stress of 12.5kPa was assumed on the rather crude basis that this is the unloading which causes 9mm heave over the length of the pile with the stiffness profile adopted. In the absence of data, the shear stress-displacement relationship shown in ***Figure 5*** was used. The results of the calculations performed by Poulos and Davis and the present authors are compared with the field data in ***Figure 9.*** As can be seen, both sets of computations give a reasonable prediction.

Extension to cases involving externally applied load

In practice (eg see Wright and Doe, 1989) it will often be necessary to determine the tension induced into a pile which is installed in swelling ground but which is also subjected to externally applied loading at the head (eg from buildings) or at the toe (base bearing or the anchoring effect of an underream). The method described herein may readily be extended to cover such situations as is shown in ***Figure 10.*** The application of a vertical load Q at the pile head or pile toe (positive when directed towards the pile) causes the LTP to shift as indicated in ***Figure 10*** in order to maintain equilibrium, and the upper-bound criterion for P_{max} based on equilibrium becomes:

$$P_{max} \leq \tfrac{1}{2}[\{\pi D \int_0^L \tau_{max}.dz\} - Q]$$

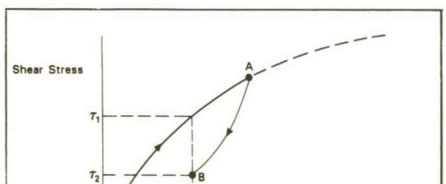

Fig 7. Stress reversal and the problem of shear stress determination for the case of a non-linear interface shear stress-displacement relationship.

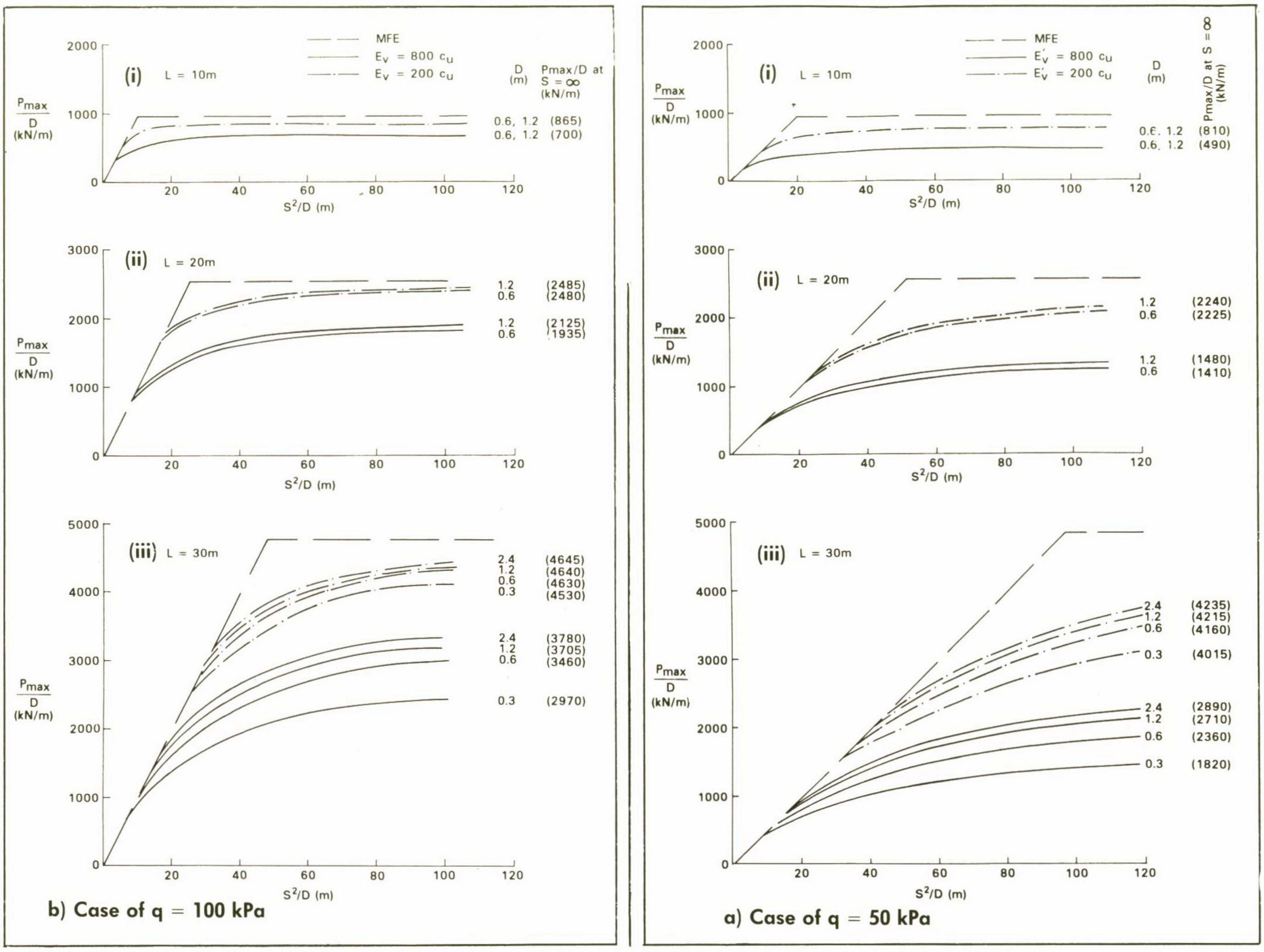

Fig 8. Illustrative example number 2. Maximum tension forces experienced by piles.

The full solution for the ATP is achieved when equilibrium and shaft stress-displacement compatibility are both attained. Caution must be exercised, however, in relation to the compatibility requirement. Since the elevation of the neutral point may change significantly when external loads are applied (as is readily observed in ***Figure 10***) it is important to consider each stage of loading or unloading in chronological sequence with proper allowances for consolidation effects and the possibility of interface stress reversal effects as described above with reference to ***Figure 7.***

Concluding note

A simple method for determining the tension force induced into a pile in swelling ground has been presented. This method produces results which appear to be consistent with the theoretical maximum force envelope (MFE) and which compare well with Poulos and Davis's elastic solution and with field data. *Examples one and two* indicate the usefulness of the present technique not only in terms of its ability to provide solutions to real engineering problems but also in its ability to allow parametric studies to be performed which help the designer understand the problem of heave induced tension.

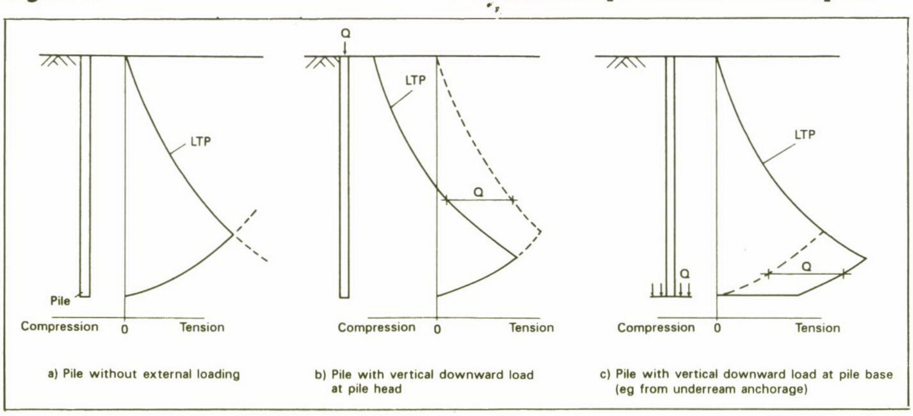

Fig 10. Limiting tension profiles for piles with and without externally applied loads.

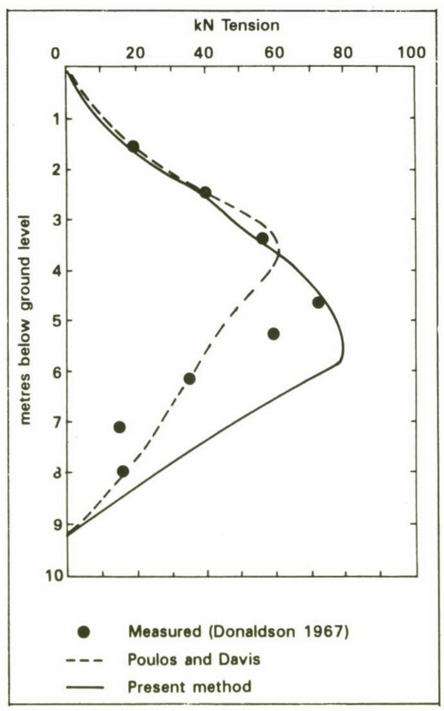

Fig 9. Comparison with field data and an elastic solution with pile-soil interface slip.

Acknowledgement
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Equipment Review

Sampler kit

A sampling and prospecting kit is now available for the Pionjär rock drill which is said to be simple to handle.

With samplers mounted on the sounding rods, it is said that samples can be taken at any depth required.

For extraction two types of mechanical rod puller are available with two or four tons pulling capacity.

Greater lifting capacity may be required when sounding and sampling and in this event a hydraulic rod puller with a capacity of 10 tons is available on request.

Reader Reply Card 2467

A geotechnician uses the Pionjär sounding kit to decide the depth of overburden down to solid bottom.

Field data

Increased sensitivity of geological measuring instruments has greatly expanded the amount of detection data and thus the risk of mistakes occurring, due to the human factor, when this data is manually recorded.

With its latest Terrameter model, the SAS 330 C, ABEM has designed the instrument to be connected to a Geomac III PC hand held computer developed especially for field use. This is completely compatible with the MS-DOS industrial standard and has an aluminium casing which is said to be completely water and dust proof.

This type of computerisation has allowed the company to avoid the creation of a hybrid measuring and data collection instrument. The customer can purchase only the functions he needs, whether it be the whole package or just the measuring instrument.

Measured data can be processed and displayed graphically on the computer display while measuring is still continuing. This can lead to a valuable saving of time in the field – as much as 20% says ABEM.

The company also claims that computerisation also leads the way to potential new applications. It is possible to monitor naturally-occurring electrical potentials in the ground, and long-term measurements of ground conductivity can also give prior warning of landslides.

Because the Geomac computer uses MS-DOS, data can be directly transferred to ABEM's own computer program for resistivity analysis, Super-VES. This program provides a graphical representation of the data and simplifies its analysis.

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Gas monitor

Geotechnical Instruments has added the handheld Infra-red Gas analyser to its range of landfill gas monitoring equipment. It has an optional data logging capability which means it can take readings unattended, essential when continuous gas profiles are required.

The system uses the principle of infra-red absorption for the quantitative analysis of methane and carbon dioxide. An electric aspirator pump draws the gas into an internal sampling chamber where infra-red light is projected through the gas to four detectors on the other side. A microprocessor calculates the amount of infra-red light absorbed at different wavelengths and determines the gas concentrations present.

Oxygen concentration measurement is available as an option and a further optional facility allows the unit to measure and automatically correct for changes in atmospheric pressure.

Based on a powerful microprocessor, the analyser will continuously read concentrations of methane and carbon dioxide over the ranges 0-10% and 0-100%, and will automatically select the most appropriate range. Methane concentrations can be expressed in terms of percentage gas or LEL and all readings are shown on a liquid crystal display.

The unit has a memory which can store up to 2000 readings together with the time, date and an identification code. These readings can then be downloaded to an IBM compatible computer for processing via an integral RS232C interface.

In addition, the operator can obtain a hard copy of the stored data on site via a small, battery-operated printer. Independent alarm levels can be selected and set in situ for any gases being sampled.

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